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Centre number

Candidate number

Surname _____

Forename(s) _____

Candidate signature _____

AS MATHEMATICS

Paper 2

Wednesday 22 May 2019

Morning

Time allowed: 1 hour 30 minutes

Materials

- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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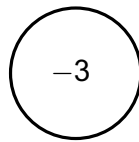
Section A

Answer **all** questions in the spaces provided.

- 1 Find the gradient of the curve $y = e^{-3x}$ at the point where it crosses the y -axis.

Circle your answer.

[1 mark]



-1

1

3

$$\frac{dy}{dx} = -3e^{-3x}$$

$$\text{At } x=0, \frac{dy}{dx} = -3.$$

- 2 Find the centre of the circle $x^2 + y^2 + 4x - 6y = 12$

Tick (✓) **one** box.

[1 mark]

(-2, -3)

(-2, 3)

(2, -3)

(2, 3)

$$\begin{aligned} x^2 + y^2 + 4x - 6y &= 12 \\ (x+2)^2 - 4 + (y-3)^2 - 9 &= 12 \\ (x+2)^2 + (y-3)^2 &= 25 \end{aligned}$$



3 It is given that $\sin \theta = -0.1$ and $180^\circ < \theta < 270^\circ$

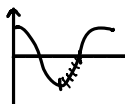
Find the exact value of $\cos \theta$

[2 marks]

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$(-0.1)^2 + \cos^2 \theta = 1 \Rightarrow 0.01 + \cos^2 \theta = 1 \Rightarrow \cos^2 \theta = 0.99$$

$$\cos \theta = \pm \sqrt{0.99} = \pm \frac{3\sqrt{11}}{10}$$

 In the region $180^\circ < \theta < 270^\circ$, $\cos \theta$ is negative, so $\cos \theta = -\frac{3\sqrt{11}}{10}$

4 Show that, for $x > 0$

$$\log_{10} \frac{x^4}{100} + \log_{10} 9x - \log_{10} x^3 \equiv 2(-1 + \log_{10} 3x)$$

[4 marks]

$$\text{LHS} = \log_{10} \frac{x^4}{100} + \log_{10} 9x - \log_{10} x^3$$

$$= \log_{10} x^4 - \log_{10} 100 + \log_{10} 9 + \log_{10} x - 3 \log_{10} x$$

$$= 4 \log_{10} x - \log_{10} 100 + \log_{10} 9 + \log_{10} x - 3 \log_{10} x$$

$$= 2 \log_{10} x - \log_{10} 10^2 + \log_{10} 3^2$$

$$= 2 \log_{10} x - 2 \log_{10} 10 + 2 \log_{10} 3$$

$$= 2(-\log_{10} 10 + 2 \log_{10} x + \log_{10} 3)$$

$$= 2(-1 + \log_{10} x + \log_{10} 3)$$

$$= 2(-1 + \log_{10} 3x) = \text{RHS}$$

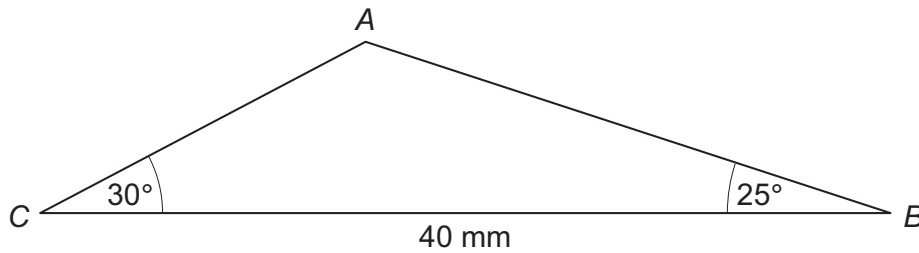
Turn over for the next question

Turn over ►



5

A triangular prism has a cross section ABC as shown in the diagram below.



$$\text{Angle } ABC = 25^\circ$$

$$\text{Angle } ACB = 30^\circ$$

$$BC = 40 \text{ millimetres.}$$

The length of the prism is 300 millimetres.

Calculate the volume of the prism, giving your answer to three significant figures.

[4 marks]

$$\text{Angle } CAB = 180 - 30 - 25 = 125^\circ$$

Using the sine rule,

$$\frac{AB}{\sin 30} = \frac{40}{\sin 125} \Rightarrow AB = \frac{40 \sin 30}{\sin 125}$$

$$\text{Area of cross-section} = \frac{1}{2} \times AB \times CB \times \sin B$$

$$= \frac{1}{2} \times \frac{40 \sin 30}{\sin 125} \times 40 \times \sin 25$$

$$= 206.368\dots$$

$$\text{Volume} = \text{Area of prism} \times \text{length of prism}$$

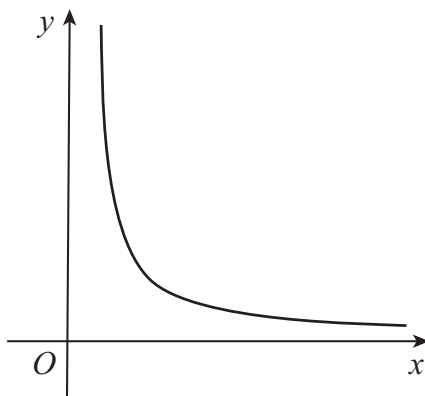
$$= 206.368\dots \times 300$$

$$= 61910.59616$$

$$= 61900 \text{ mm}^3 \text{ (3.s.f)}$$



- 6 A curve has equation $y = \frac{2}{x\sqrt{x}}$



The region enclosed between the curve, the x -axis and the lines $x = 1$ and $x = a$ has area 3 units.

Given that $a > 1$, find the value of a .

Fully justify your answer.

[5 marks]

$$y = 2x^{-\frac{3}{2}}$$

$$3 = \int_1^a 2x^{-\frac{3}{2}} dx$$

$$3 = [-4x^{-\frac{1}{2}}]_1^a$$

$$3 = [(-4a^{-\frac{1}{2}}) - (-4)]$$

$$3 = -4a^{-\frac{1}{2}} + 4$$

$$\frac{4}{\sqrt{a}} = 1 \Rightarrow \sqrt{a} = 4$$

$$a = 16$$

Turn over ►



7 The points $A(a, 3)$ and $B(10, 6)$ lie on a circle.

AB is a diameter of the circle and passes through the point $(2, 4)$

The circle has equation

$$(x - c)^2 + (y - d)^2 = e$$

where c , d and e are rational numbers.

Find the values of a , c , d and e .

[6 marks]

Gradient of line from $(2, 4)$ to B : $\frac{6-4}{10-2} = \frac{2}{8} = \frac{1}{4}$

Gradient of line from $(2, 4)$ to A : $\frac{4-3}{2-a} = \frac{1}{2-a}$

$$\frac{1}{4} = \frac{1}{2-a}$$

$$2-a = 4 \Rightarrow a = -2$$

Midpoint of $AB = \left(\frac{10+a}{2}, \frac{6+3}{2}\right) = \left(\frac{10+a}{2}, 4.5\right)$

$$= (4, 4.5) \text{ by substituting in } a = -2.$$

So the centre of the circle is at $(4, 4.5)$ and since the

centre is (c, d) : $c = 4$

$$d = 4.5$$

Since $(10, 6)$ lies on the circle: $e = (10-c)^2 + (6-d)^2$

$$e = (6)^2 + (1.5)^2$$

$$e = 38.25$$

$$\therefore a = -2$$

$$c = 4$$

$$d = 4.5$$

$$e = 38.25$$



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Turn over for the next question

Turn over ►



8 A curve has equation

$$y = x^3 + px^2 + qx - 45$$

The curve passes through point $R(2, 3)$

The gradient of the curve at R is 8

8 (a) Find the value of p and the value of q .

[5 marks]

$$y = x^3 + px^2 + qx - 45$$

$$\text{Since the curve passes through } (2, 3): 3 = 2^3 + p(2)^2 + q(2) - 45$$

$$3 = 8 + 4p + 2q - 45$$

$$4p + 2q = 40 \quad \textcircled{1}$$

$$\frac{dy}{dx} = 3x^2 + 2px + q$$

$$\text{At } (2, 3), \frac{dy}{dx} = 8$$

$$3(2)^2 + 2p(2) + q = 8$$

$$12 + 4p + q = 8$$

$$4p + q = -4 \quad \textcircled{2}$$

$$\textcircled{1} - \textcircled{2}: 4p + 2q - (4p + q) = 40 - (-4)$$

$$q = 44$$

$$\text{Substitute } q = 44 \text{ into } \textcircled{1}: 4p + 2(44) = 40$$

$$4p = -48$$

$$p = -12$$

$$\therefore p = -12, q = 44$$



- 8 (b) Calculate the area enclosed between the normal to the curve at R and the coordinate axes.

[5 marks]

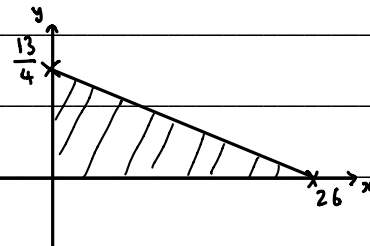
Gradient of normal to curve at R is: $-1 \times \frac{1}{8} = -\frac{1}{8}$.

Equation of normal: $(y-3) = -\frac{1}{8}(x-2)$

$$y = -\frac{1}{8}x + \frac{13}{4}$$

Intersects x axis when $-\frac{1}{8}x + \frac{13}{4} = 0 \Rightarrow x = 26$

Intersects y axis when $y = 0 + \frac{13}{4} \Rightarrow y = \frac{13}{4}$



Area required is the shaded area in the diagram.

$$\text{Area} = \frac{1}{2} \times 26 \times \frac{13}{4} = \frac{169}{4}$$

Turn over ►



9 A curve C has equation $y = f(x)$ where

$$f(x) = (x - 2)(x - 3)^2$$

9 (a) Find the exact coordinates of the turning points of C.

Determine the nature of each turning point.

Fully justify your answer.

[8 marks]

$$f(x) = (x-2)(x-3)^2$$

$$f(x) = (x-2)(x^2 - 6x + 9)$$

$$f(x) = x^3 - 6x^2 + 9x - 2x^2 + 12x - 18$$

$$f(x) = x^3 - 8x^2 + 21x - 18$$

$$f'(x) = 3x^2 - 16x + 21$$

$$\text{A turning point occurs when } f'(x) = 0: \quad 3x^2 - 16x + 21 = 0$$

$$3x^2 - 9x - 7x + 21 = 0$$

$$3x(x-3) - 7(x-3) = 0$$

$$(3x-7)(x-3) = 0$$

$$x = \frac{7}{3} \quad \text{or} \quad x = 3$$

$$\text{When } x = \frac{7}{3}, \quad y = \left(\frac{7}{3}\right)^3 - 8\left(\frac{7}{3}\right)^2 + 21\left(\frac{7}{3}\right) - 18 = \frac{4}{27}$$

$$\text{When } x = 3, \quad y = (3)^3 - 8(3)^2 + 21(3) - 18 = 0$$

So, turning points occur at $\left(\frac{7}{3}, \frac{4}{27}\right)$ and $(3, 0)$.

To determine nature, we look at sign of $f''(x)$.

$$f''(x) = 6x - 16$$

$$f''\left(\frac{7}{3}\right) = 6\left(\frac{7}{3}\right) - 16 = -2 < 0 \quad \text{so maximum at } \left(\frac{7}{3}, \frac{4}{27}\right)$$

$$f''(3) = 6(3) - 16 = 2 > 0 \quad \text{so minimum at } (3, 0)$$



- 9 (b) State the coordinates of the turning points of the curve

$$y = f(x + 1) - 4$$

[2 marks]

The graph has shifted one unit in the negative x direction
and 4 units in the negative y direction.

Turning points: $\left(\frac{7}{3} - 1, \frac{4}{27} - 4\right) = \left(\frac{4}{3}, -\frac{104}{27}\right)$

$$(3 - 1, 0 - 4) = (2, -4).$$

So the turning points are $\left(\frac{4}{3}, -\frac{104}{27}\right), (2, -4)$.

Turn over for the next question

Turn over ►



- 10** As part of an experiment, Zena puts a bucket of hot water outside on a day when the outside temperature is 0°C .

She measures the temperature of the water after 10 minutes and after 20 minutes. Her results are shown below.

Time (minutes)	10	20
Temperature (degrees Celsius)	30	12

Zena models the relationship between θ , the temperature of the water in $^{\circ}\text{C}$, and t , the time in minutes, by

$$\theta = A \times 10^{-kt}$$

where A and k are constants.

- 10 (a)** Using $t = 0$, explain how the value of A relates to the experiment.

[1 mark]

$$\text{At } t=0, \theta = A \times 10^{-0} = A$$

So the temperature of the water at the start is A .

- 10 (b)** Show that

$$\log_{10} \theta = \log_{10} A - kt$$

[1 mark]

$$\log_{10} \theta = \log_{10} (A \times 10^{-kt})$$

$$\log_{10} \theta = \log_{10} A + \log_{10} 10^{-kt}$$

$$\log_{10} \theta = \log_{10} A - kt \log_{10} 10$$

$$\log_{10} \theta = \log_{10} A - kt$$

- 10 (c)** Using Zena's results, calculate the values of A and k .

[4 marks]

$$\text{At } t=10, \theta=30: \log_{10} 30 = \log_{10} A - 10k \Rightarrow \log_{10} A = 10k + \log_{10} 30$$

$$\text{At } t=20, \theta=12: \log_{10} 12 = \log_{10} A - 20k \Rightarrow \log_{10} A = 20k + \log_{10} 12$$



$$10k + \log_{10} 30 = 20k + \log_{10} 12$$

$$10k = \log_{10} 30 - \log_{10} 12$$

$$10k = \log_{10} \left(\frac{30}{12}\right)$$

$$k = \frac{1}{10} \log_{10} \left(\frac{5}{2}\right)$$

$$k = 0.039794\dots$$

$$k = 0.0398$$

$$\theta = A \times 10^{-kt}$$

$$A = \frac{\theta}{10^{-kt}} = \frac{30}{10^{-10(0.0398)}} = 75.01036\dots \Rightarrow A = 75$$

$$\therefore k = 0.0398, A = 75$$

- 10 (d) Zena states that the temperature of the water will be less than 1°C after 45 minutes. Determine whether the model supports this statement.

[3 marks]

$$\theta = 75 \times 10^{-0.0398(45)}$$

$$= 1.21 > 1$$

Therefore the model does not support Zena's statement.

- 10 (e) Explain why Zena's model is unlikely to accurately give the value of θ after 45 minutes.

[1 mark]

After 45 minutes the outside temperature is unlikely to still be 0° .

Turn over ►



Section B

Answer **all** questions in the spaces provided.

- 11** A survey is undertaken to find out the most popular political party in London.
The first 1100 available people from London are surveyed.
Identify the name of this type of sampling.

Circle your answer.

simple random



opportunity

stratified

quota

[1 mark]

- 12** Manny is studying the price and number of pages of a random sample of books.
He calculates the value of the product moment correlation coefficient between the price and number of pages in each book as 1.05
Which of the following best describes the value 1.05?

Tick (✓) **one** box.

definitely correct

probably correct

probably incorrect

definitely incorrect

[1 mark]



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13 Denzel wants to buy a car with a propulsion type **other than** petrol or diesel.

He takes a sample, from the Large Data Set, of the CO₂ emissions, in g/km, of cars with one particular propulsion type.

The sample is as follows

82 13 96 49 96 92 70 81

13 (a) Using your knowledge of the Large Data Set, state which propulsion type this sample is for, giving a reason for your answer.

[2 marks]

Electric / petrol

It is the only propulsion type with 8 values.

13 (b) Calculate the mean of the sample.

[1 mark]

$$\frac{(82 + 13 + 96 + 49 + 96 + 92 + 70 + 81)}{8} = \frac{579}{8} = 72.375$$

$$= 72.4$$

13 (c) Calculate the standard deviation of the sample.

[1 mark]

$$\left(\frac{1}{7} \left[(82-72.4)^2 + (13-72.4)^2 + (96-72.4)^2 + (49-72.4)^2 + (96-72.4)^2 + (92-72.4)^2 + (70-72.4)^2 + (81-72.4)^2 \right] \right)^{\frac{1}{2}}$$

$$= 28.650... = 28.7$$

Here we divided by one less data point since the data is from a larger population.



13 (d) Denzel claims that the value 13 is an outlier.

13 (d) (i) Any value more than 2 standard deviations from the mean can be regarded as an outlier.

Verify that Denzel's claim is correct.

[1 mark]

$$72.4 - 2(28.7) = 15$$

Since $13 < 15$, the value 13 is an outlier and Denzel is correct.

13 (d) (ii) State what effect, if any, removing the value 13 from the sample would have on the standard deviation.

[1 mark]

The standard deviation would decrease.

Turn over for the next question

Turn over ►



14 A probability distribution is given by

$$P(X = x) = c(4 - x), \text{ for } x = 0, 1, 2, 3$$

where c is a constant.

14 (a) Show that $c = \frac{1}{10}$

[2 marks]

$$P(X=0) + P(X=1) + P(X=2) + P(X=3) = 1$$

$$c(4-0) + c(4-1) + c(4-2) + c(4-3) = 1$$

$$4c + 3c + 2c + c = 1$$

$$10c = 1$$

$$c = \frac{1}{10}$$

14 (b) Calculate $P(X \geq 1)$

[2 marks]

$$P(X \geq 1) = 1 - P(X=0)$$

$$= 1 - 4c$$

$$= 1 - \frac{4}{10}$$

$$= 0.6$$



15 Two independent events, A and B , are such that

$$P(A) = 0.2$$

$$P(A \cup B) = 0.8$$

15 (a) (i) Find $P(B)$

[4 marks]

$$\text{For independent events: } P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = P(A) + P(B) - P(A)P(B)$$

$$P(B) = \frac{P(A \cup B) - P(A)}{1 - P(A)}$$

$$P(B) = \frac{0.8 - 0.2}{1 - 0.2}$$

$$P(B) = \frac{0.6}{0.8} = 0.75$$

15 (a) (ii) Find $P(A \cap B)$

[1 mark]

$$P(A \cap B) = P(A)P(B) = 0.2 \times 0.75 = 0.15$$

15 (b) State, with a reason, whether or not the events A and B are mutually exclusive.

[1 mark]

They are not mutually exclusive since $P(A \cap B) \neq 0$.

Turn over ►



16 Andrea is the manager of a company which makes mobile phone chargers.

In the past, she had found that 12% of all chargers are faulty.

16 (a) Andrea decides to move the manufacture of chargers to a different factory.

Andrea tests 60 of the new chargers and finds that 4 chargers are faulty.

Investigate, at the 10% level of significance, whether the proportion of faulty chargers has reduced.

[7 marks]

$$H_0: p = 0.12$$

$$H_1: p < 0.12$$

$$X \sim B(60, 0.12)$$

$$P(X \leq 4) = 0.139$$

Since $0.139 > 0.1$ accept H_0 . There is insufficient evidence to suggest that the proportion of faulty chargers has decreased.



- 16 (b)** State, **in context**, two assumptions that are necessary for the distribution that you have used in part (a) to be valid.

[2 marks]

A charger being faulty is independent of any other charger being faulty.

The probability of a faulty charger is fixed.

END OF QUESTIONS



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